Gas Path Analysis (GPA)

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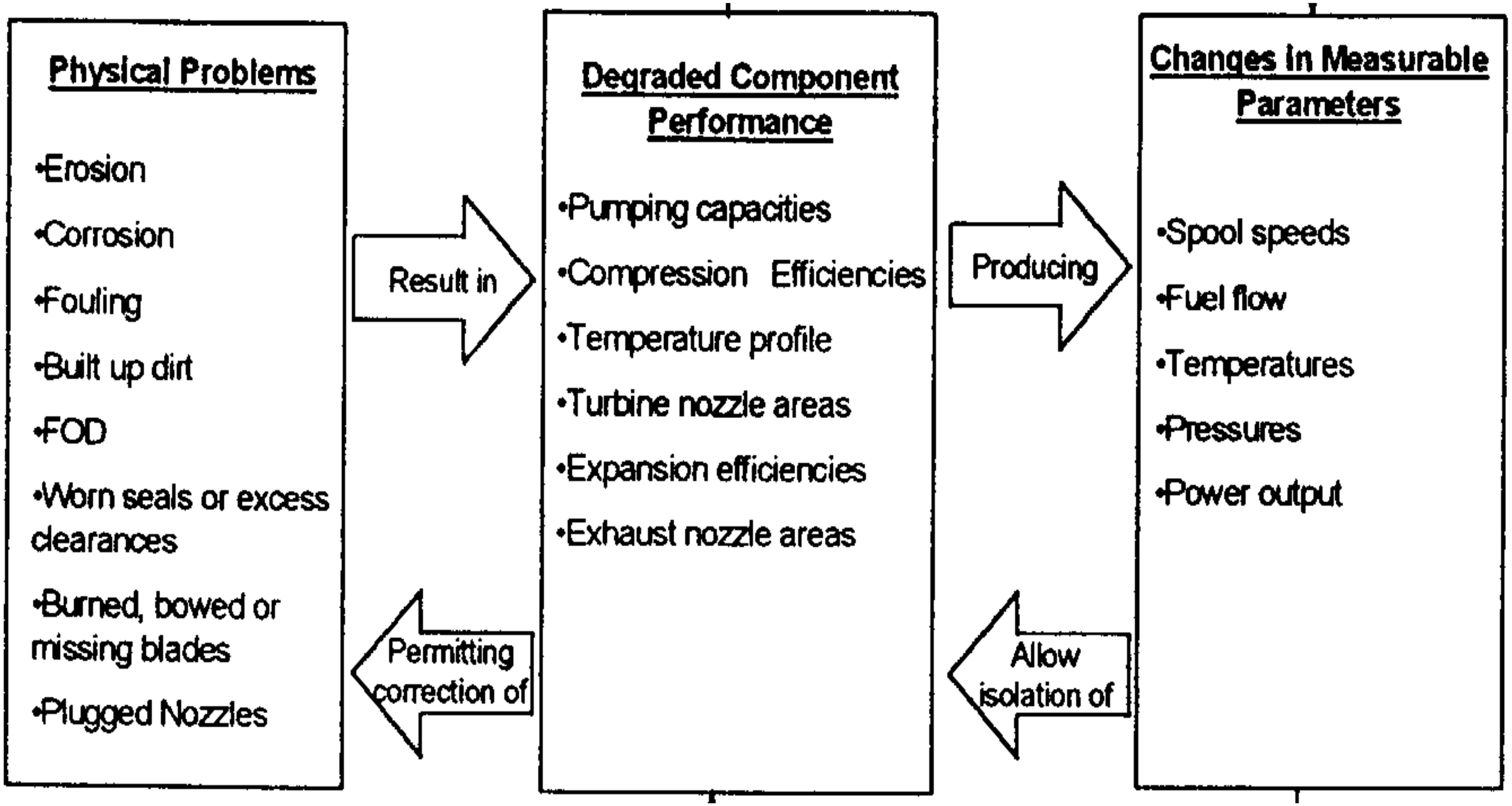
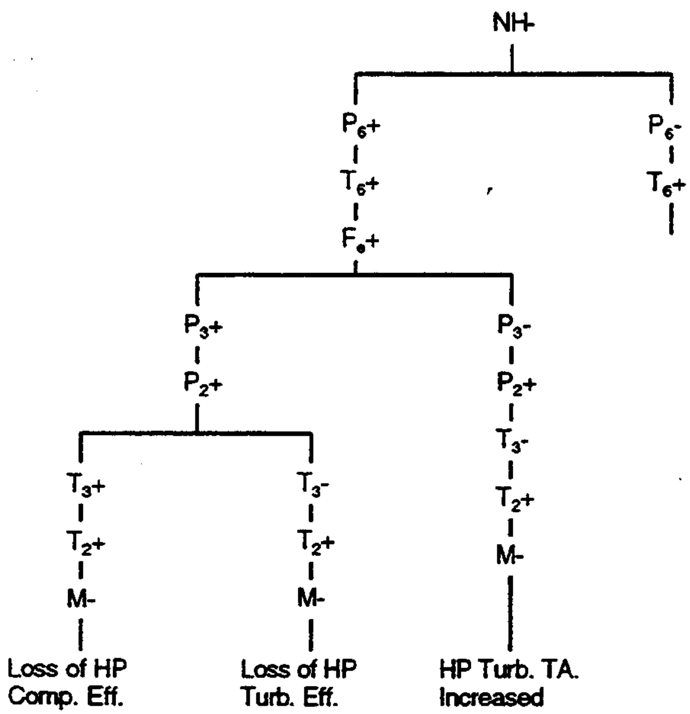


(Courtesy of Urban L. A. AGARD-CP-165, 1975)

Engine fault and parameter relationship

Fault tree & Fault Matrix



Type	Fault	TIT	SHP	m_f	CPR	Vibration	Indication
Turbine (Generator)	Rotor Damage	↑	↑	↑	↑	Yes	η_t Low
	Nozzle Erosion	↑	↑	↑	↓	No	$m\sqrt{T_3}/P_3$ High
Turbine (Power)	Rotor Damage	0	↓	0	0	Yes	η_t Low, EGT High
	Nozzle Erosion	↓	↓	↓	↓	No	$m\sqrt{T_4}/P_4$ High
Compressor	F.O.D.	↑	↓	↑	↓	Yes	η_c Low, m_1 Low
	Dirty	↓	↓	↓	↓	No	η_c Low

Performance Simulation & Diagnostics

Simulation

$$\vec{z} = h(\vec{x}) + \vec{b} + \vec{v}$$

measurements | **Performance parameters** | **biases** | **noises**

Diagnosis

Direct matrix inverse approach

Engine model: $\vec{z} = h(\vec{x})$

0 – Nominal diagnostic point

Expansion: $$\vec{z} = h(\vec{x}_0) + \frac{\partial h(\vec{x})}{\partial x}(\vec{x} - \overrightarrow{x_0}) + \mathrm{HOT}$$

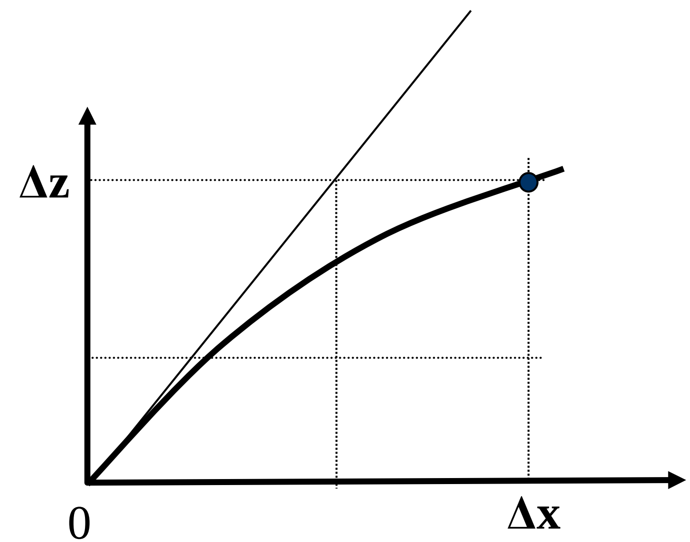
$$\vec{z} = \overrightarrow{z_0} + \frac{\partial z}{\partial x}(\vec{x} - \overrightarrow{x_0})$$

Linear engine model: $$\Delta\vec{z} = H \cdot \Delta\vec{x}$$

(ICM)

Solution: $$\Delta\vec{x} = H^{-1} \cdot \Delta\vec{z}$$

(FCM)



Gas Path Analysis (GPA)

Linear engine model: $\Delta\vec{z} = H \cdot \Delta\vec{x}$

$$\begin{pmatrix} \Delta z_1 \\ \cdots \\ \Delta z_M \end{pmatrix} = \begin{pmatrix} \frac{\partial h_1(\vec{x})}{\partial x_1} & \cdots & \frac{\partial h_1(\vec{x})}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_M(\vec{x})}{\partial x_1} & \cdots & \frac{\partial h_M(\vec{X})}{\partial x_N} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_1 \\ \cdots \\ \Delta x_N \end{pmatrix}$$

Gas Path Analysis (GPA)

Non-dimensional linear engine model:

$$\begin{pmatrix} \Delta z_1/z_1 \\ \cdots \\ \Delta z_M/z_M \end{pmatrix} = \begin{pmatrix} \dfrac{\Delta z_1/z_1}{\Delta x_1/x_1} & \cdots & \dfrac{\Delta z_1/z_1}{\Delta x_N/x_N} \\ \vdots & \ddots & \vdots \\ \dfrac{\Delta z_M/z_M}{\Delta x_1/x_1} & \cdots & \dfrac{\Delta z_M/z_M}{\Delta x_N/x_N} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_1/x_1 \\ \cdots \\ \Delta x_N/x_N \end{pmatrix}$$

Gas Path Analysis (GPA)

Inverse of a square matrix:

$$\Delta\vec{z} = H \cdot \Delta\vec{x} \Longleftrightarrow \Delta\vec{x} = H^{-1} \cdot \Delta\vec{z}$$

$$H = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$H^{-1} = \frac{1}{|H|} \cdot \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Determinant of $H = |a_{11} * a_{22} - a_{21} * a_{12}|$

Adjoint of H

GPA solution for different sensor numbers:

Sensor number > number of health parameters:

$$\Delta\vec{z} = H \cdot \Delta\vec{x} \quad \Longleftrightarrow \quad \begin{pmatrix} \Delta z_1 \\ \Delta z_2 \\ \Delta z_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} \cdot \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \end{pmatrix}$$

$$H^T \cdot \Delta\vec{z} = H^T \cdot H \cdot \Delta\vec{x}$$

$$(H^T \cdot H)^{-1} \cdot H^T \cdot \Delta\vec{z} = \Delta\vec{x}$$

$$\Delta\vec{x} = H^\# \cdot \Delta\vec{z}$$

(Pseudo inverse matrix)

GPA solution for different sensor numbers:

Sensor number = number of health parameters:

$$\Delta\vec{x} = H^{-1} \cdot \Delta\vec{z}$$

Sensor number > number of health parameters:

$$\Delta\vec{x} = H^{\#} \cdot \Delta\vec{z} = (H^T \cdot H)^{-1} \cdot H^T \cdot \Delta z$$

Sensor number < number of health parameters:

$$\Delta\vec{x} = H^{\#} \cdot \Delta\vec{z} = H^T \cdot (H \cdot H^T)^{-1} \cdot \Delta z$$

The result is best in a least-squares sense

Gas Path Analysis (GPA)

$$\Delta\vec{z} = H \cdot \Delta\vec{x}$$

Assumptions for GPA:

- **A set of accurate measurement deltas (Δz) is available**

  - repeatable, free of measurement noise & bias

- **The linear model represents engine performance accurately around a reference point**

- **The ICM (*H*) is invertible**

Potential capabilities of linear GPA:

- **Simple**
- **Fast**
- **Fault detection**
- **Fault isolation**
- **Fault quantification**
- **Deal with multiple faults**

Challenges for linear GPA:

- **Data repeatability**
- **Non-linearity**
- **Selection of measurements**
- **Smearing effect**

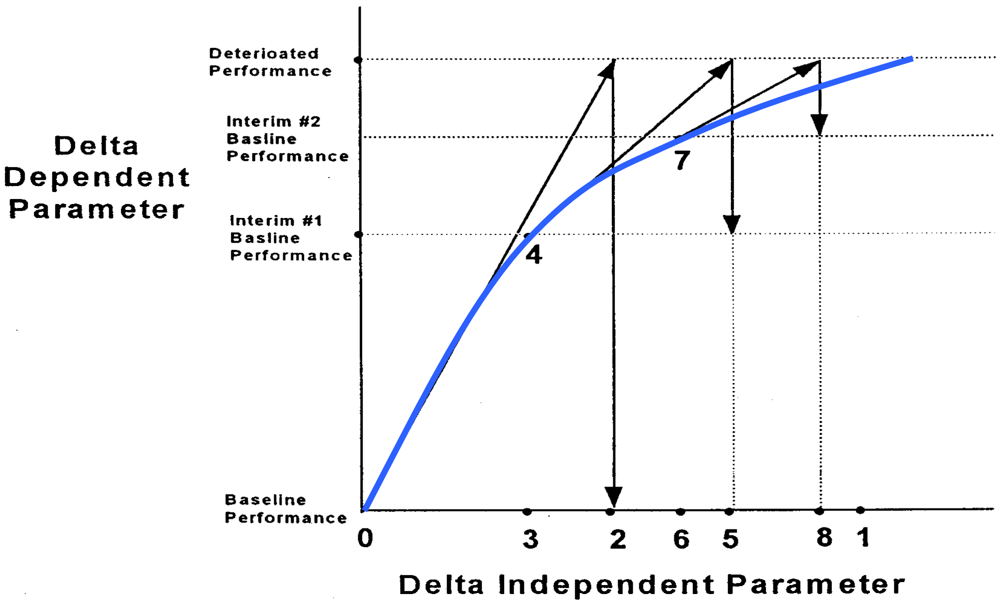
Solutions to Challenges for linear GPA:

- **Measurement noise** — **- noise filter**
- **Sensor fault** — **- sensor diagnosis to exclude faulty sensors**
- **Data uncertainty due to other factors – changing ambient and operating conditions, etc.** — **- data corrections**

Solutions to Challenges for linear GPA:

- Non-linearity of engine performance

Non-linear GPA



Convergence of non-linear GPA:

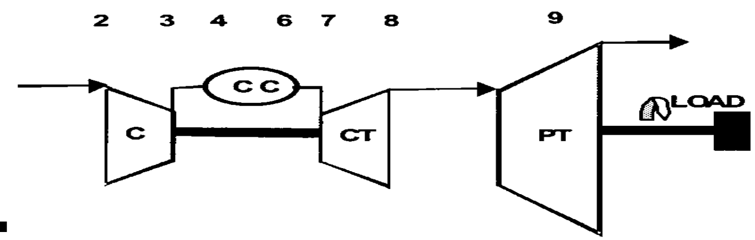
- Under-relaxation
- Convergence criteria:

$$\Delta\vec{z}_{sum}=\sum_j^M \left|\Delta z_{meas_j} - \Delta z_{cal_j}\right| < \delta$$

Selection of instrumentation sets

Solutions to Challenges for linear GPA:

- Selection of measurements



CASE STUDY: A							ENGINE: RR AVON (2 - SHAFT)						
HANDLE: SHP							FAULT LEVEL: FI = 1%						
VAR.	1	2	3	4	5	6	7	8	9	10	11	12	13
N1	x	x	x	x	x	x	x					x	x
P3						x							x
T3	x												x
P6					x								x
T6		x						x					x
WFE							x				x		x
P8													x
T8			x						x				x
P9								x	x	x	x	x	x
T9				x						x			x
SHP													
ETAC1	x	x	x	x	x	x	x	x	x	x	x	x	x
NDMC1	x	x	x	x	x	x	x	x	x	x	x	x	x
ETAT1													
NDMT1													
ETATPT													
NDMPT													
RMS													
LPGA	0.767	0.652	0.629	0.621	0.724	0.730	0.575	2.213	2.124	2.122	2.086	0.978	0.678
NLPGA	0.108	0.097	0.123	0.124	0.219	0.214	0.108	0.091	0.086	0.197	0.118	1.523	0.053

Selection of instrumentation sets

Solutions to Challenges for linear GPA:

- Selection of measurements

- **Availability**
- **Number of sought faults & health parameters**
- **Sensitivity**
- **Correlation**
- **Redundancy**

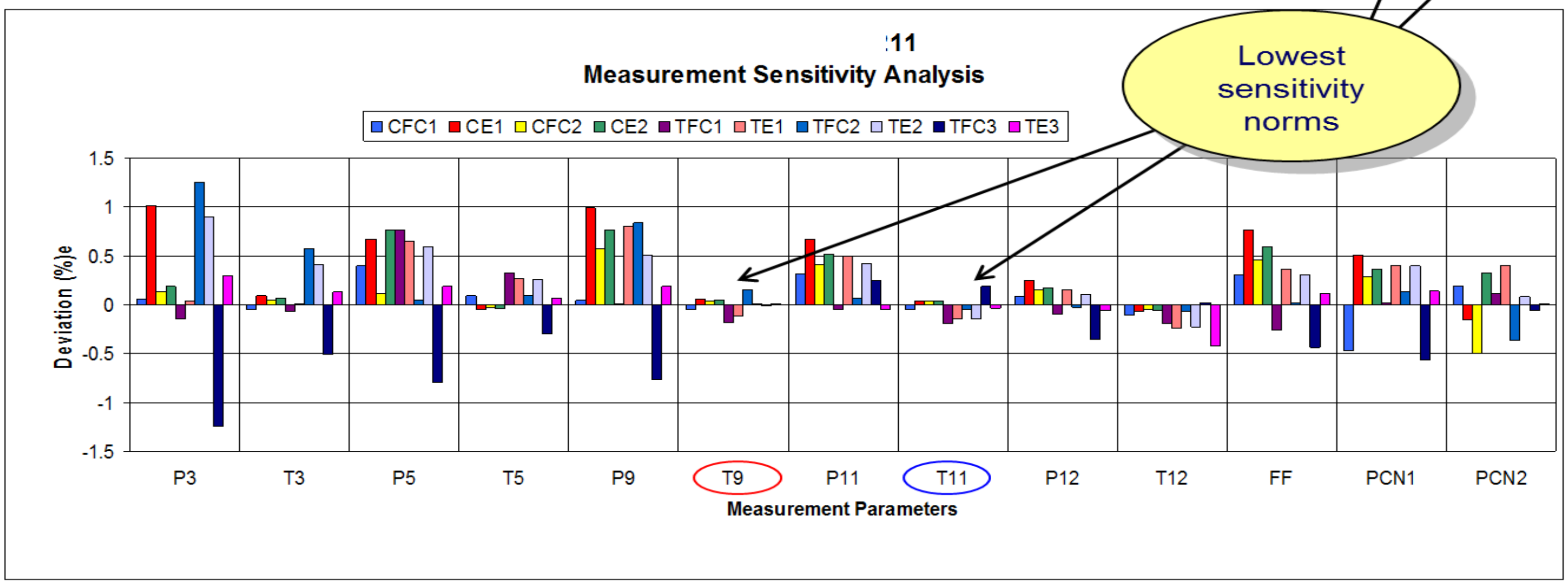
Gas Path Analysis (GPA)

- Selection of measurements - sensitivity

$$s_{i,j} = \frac{\partial \vec{z}_i / \vec{z}_i}{\partial \vec{x}_j / \vec{x}_j} \qquad \left\| s_i \right\| = \sqrt{\sum_{j=1}^{N} (s_{i,j})^2} = \sqrt{(s_{i,1})^2 + \ldots + (s_{i,N})^2}$$

Measurement	P3	P9	P5	FF	P11	PCN1	T3	PCN2	T5	T12	P12	T11	T9
Sensitivity Norm	2.2627	2.0301	1.8033	1.3185	1.2294	1.1742	0.9020	0.8499	0.6052	0.5909	0.5458	0.3506	0.2908

SENSITIVITY NORMS



SENSITIVITY ANALYSIS BAR CHART

Gas Path Analysis (GPA)

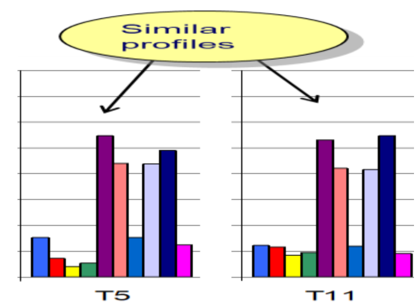
- Selection of measurements - correlation

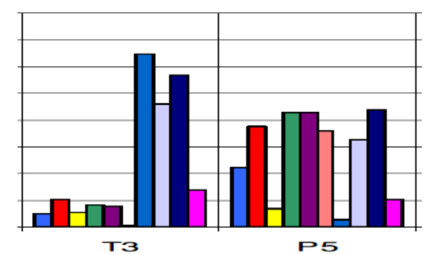
$$n_{i,j} = s_{i,j} / \|s_i\| \Rightarrow Q = P \cdot P^T \qquad \|c_i\| = \sqrt{\sum_{h=1}^{L} (c_{i,h})^2} = \sqrt{(c_{i,1})^2 + \ldots + (c_{i,L})^2}$$

Highest correlation

	P3	T3	P5	T5	P9	T9	P11	T11	P12	T12	FF			
P3	1	0.9295	0.5920	0.4816	0.8221	0.4940	0.3761	-0.4570	0.6782	-0.3915	[illegible]	[illegible]	[illegible]	2.3010
T3	0.9295	1	0.4653	0.5341	0.7059	0.4987	0.1789	-0.5242	0.5267	-0.3642	0.4400	0.6226	-0.2241	2.1121
P5	0.5920	0.4653	1	0.7496	0.7803	-0.2385	0.6211	-0.7063	0.7303	-0.6414	0.7316	0.6952	0.3863	2.4066
T5	0.4816	0.5341	0.7496	1	0.4816	-0.4600	0.1797	-0.9936	0.3874	-0.6615	0.2472	0.4576	0.3463	2.1261
P9	0.8221	0.7059	0.7803	0.4816	1	0.2957	0.7218	-0.4251	0.8101	-0.5065	0.8648	0.8503	-0.0392	2.4878
T9	0.4940	0.4987	-0.2385	-0.4600	0.2957	1	0.0929	0.4767	0.1619	0.2313	0.2597	0.1991	-0.5957	1.6206
P11	0.3761	0.1789	0.6211	0.1797	0.7218	0.0929	1	-0.0901	0.5769	-0.4370	0.8186	0.5104	0.1050	1.8960
T11	-0.4570	-0.5242	-0.7063	-0.9936	-0.4251	0.4767	-0.0901	1	-0.3747	0.5970	-0.1908	-0.4359	-0.3612	2.0590
P12	0.6782	0.5267	0.7303	0.3874	0.8101	0.1619	0.5769	-0.3747	1	-0.2205	0.9011	0.7732	0.1037	2.2462
T12	-0.3915	-0.3642	-0.6414	-0.6615	-0.5065	0.2313	-0.4370	0.5970	-0.2205	1	-0.4026	-0.4203	-0.2537	1.8558
FF	0.6406	0.4400	0.7316	0.2472	0.8648	0.2597	0.8186	-0.1908	0.9011	-0.4026	1	0.7310	0.0573	2.2878
PCN1	0.7072	0.6226	0.6952	0.4576	0.8503	0.1991	0.5104	-0.4359	0.7732	-0.4203	0.7310	1	0.0039	2.2589
PCN2	-0.2336	-0.2241	0.3863	0.3463	-0.0392	-0.5957	0.1050	-0.3612	0.1037	-0.2537	0.0573	0.0039	1	1.3965

CORRELATIONS MATRIX

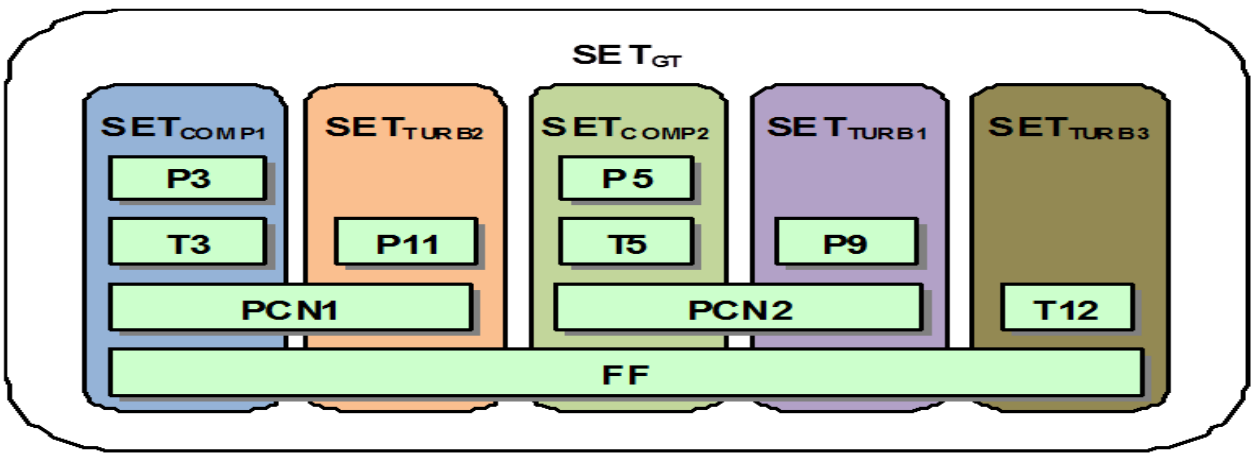




Gas Path Analysis (GPA)

- Selection of measurements – sub-set & redundancy

No.	Final Measurement Parameter	Ind.
1	IP Compressor Exit Pressure	P3
2	IP Compressor Exit Temperature	T3
3	HP Compressor Exit Pressure	P5
4	HP Compressor Exit Temperature	T5
5	HP Turbine Exit Pressure	P9
6	IP Turbine Exit Pressure	P11
7	Power Turbine Exit Temperature	T12
8	Fuel Flow	FF
9	IP Compressor Speed	PCN1
10	HP Compressor Speed	PCN2

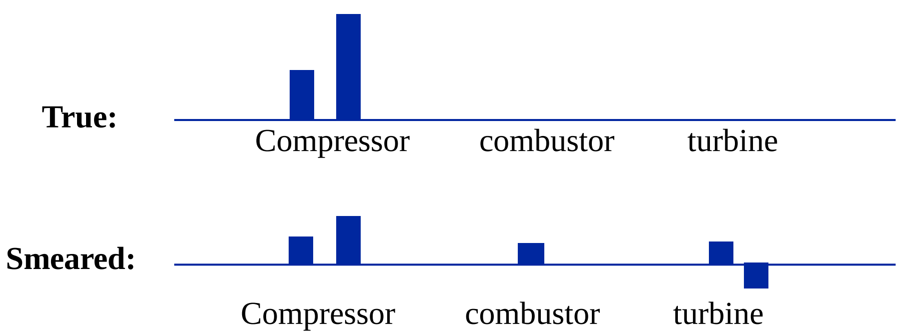


MEASUREMENTSUBSET DIAGRAM

Gas Path Analysis (GPA)

Solutions to Challenges for linear GPA:

- Smearing effect / fault isolation



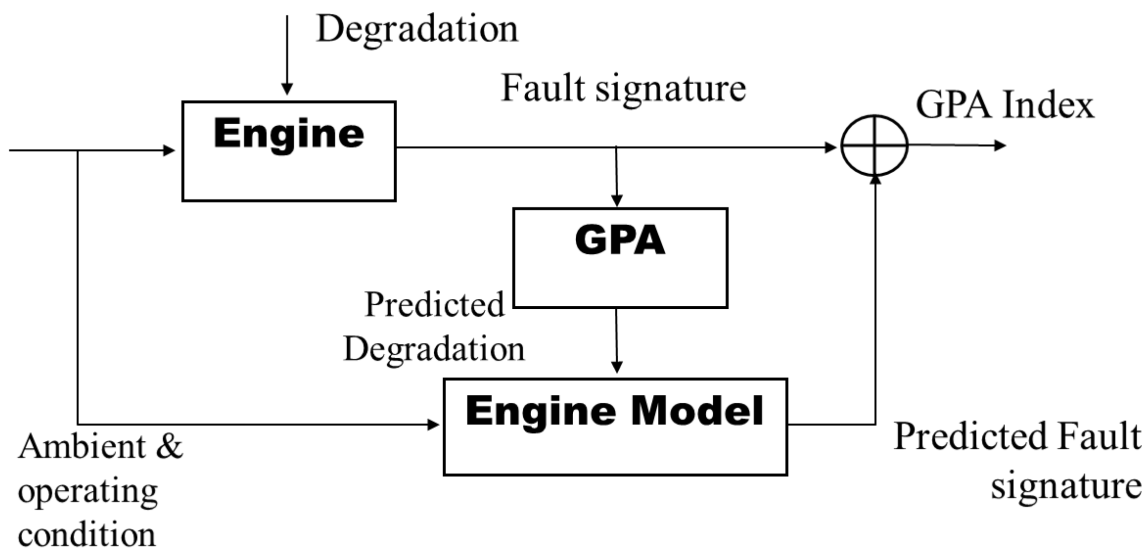
- **Fault isolation using "Component Fault Cases (CFC)" concept**

Component Fault Case	Pre-defined faulty components
CFC1	Compressor
CFC2	Burner
CFC3	Turbine 1
CFC4	Turbine 2
CFC5	Compressor + Burner
CFC6	Compressor + Turbine 1
CFC7	Compressor + Turbine 2
CFC8	Burner + Turbine 1
CFC9	Burner +Turbine 2
CFC10	Turbine 1 + TURBINE 2

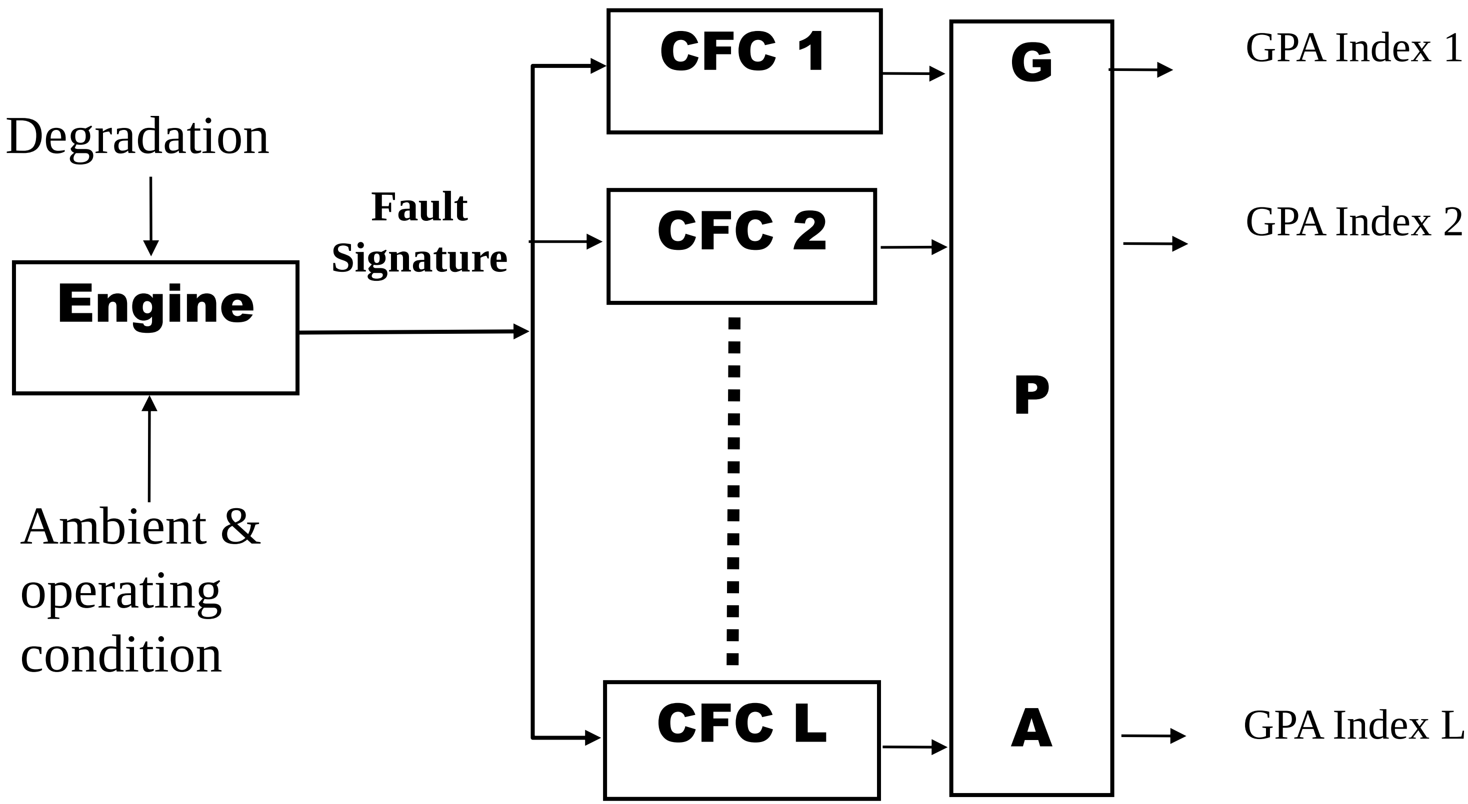
Gas Path Analysis (GPA)

GPA Index

$$GPA \quad Index = \frac{1}{1+\varepsilon} \qquad \varepsilon = \sum_{i=1}^{N} \left| \frac{\Delta z_i}{z_0} - \frac{\Delta z_{0,i}}{z_0} \right|$$

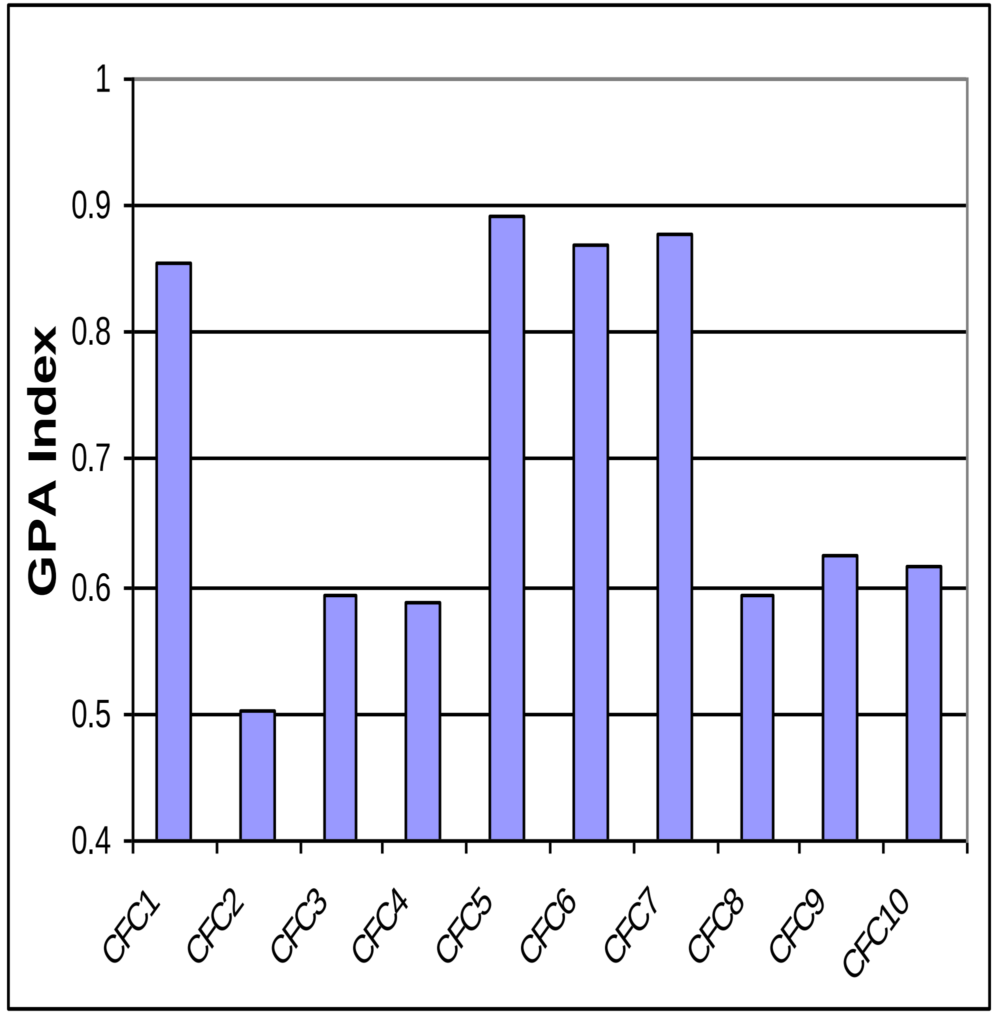


GPA Index and fault isolation



GPA Index and fault isolation

Component Fault Case	Pre-defined faulty components
CFC1	Compressor
CFC2	Burner
CFC3	Turbine 1
CFC4	Turbine 2
CFC5	Compressor + Burner
CFC6	Compressor + Turbine 1
CFC7	Compressor + Turbine 2
CFC8	Burner + Turbine 1
CFC9	Burner +Turbine 2
CFC10	Turbine 1 + TURBINE 2



GPA Index and fault isolation

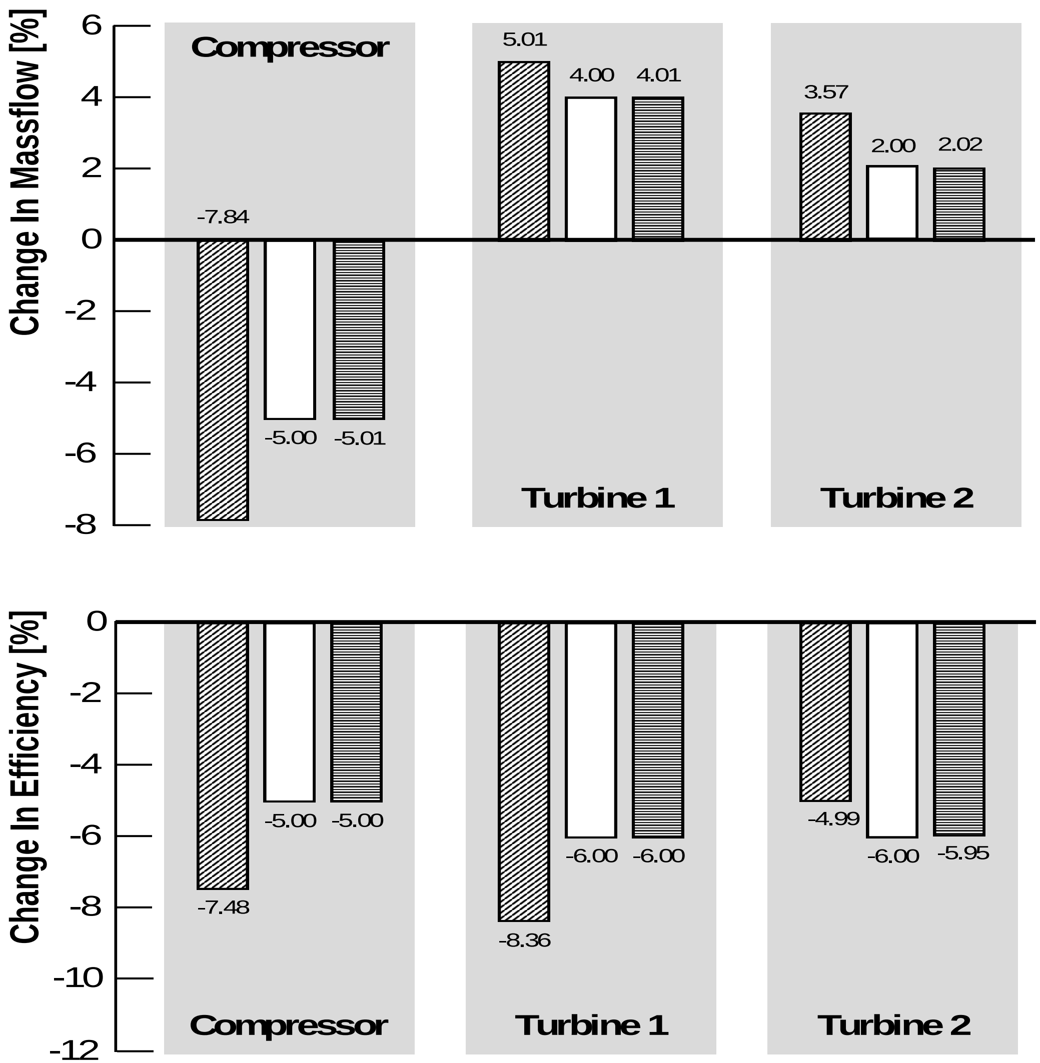
Predicted engine degradation (Implemented fault: compressor $\Delta\eta_c$=-2.5, $\Delta\Gamma_c$=-4.5)

		CFC1	CFC5	CFC6	CFC7
Compressor	$\Delta\eta_c$	-2.479	-2.544	-2.560	-2.616
	$\Delta\Gamma_c$	-3.577	-3.633	-3.622	-3.766
Burner	$\Delta\eta_b$		0.375		
Turbine 1	$\Delta\eta_{t1}$			0.222	
	$\Delta\Gamma_{t1}$			0.252	
Turbine 2	$\Delta\eta_{t2}$				0.296
	$\Delta\Gamma_{t2}$				0.089

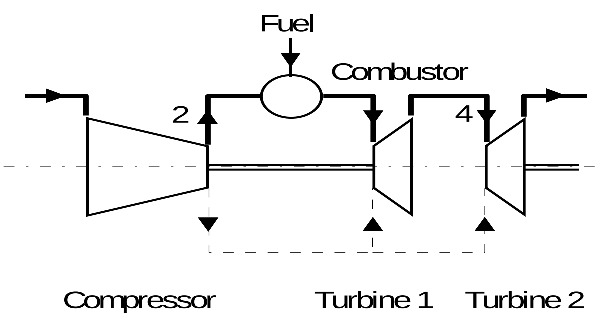
Comparison between linear & non-linear GPAs

	Linear GPA	Non-linear GPA
Accuracy:	Low	Higher in general
Computation time:	Short	Slightly Longer
Convergence	No problem in general	May diverge

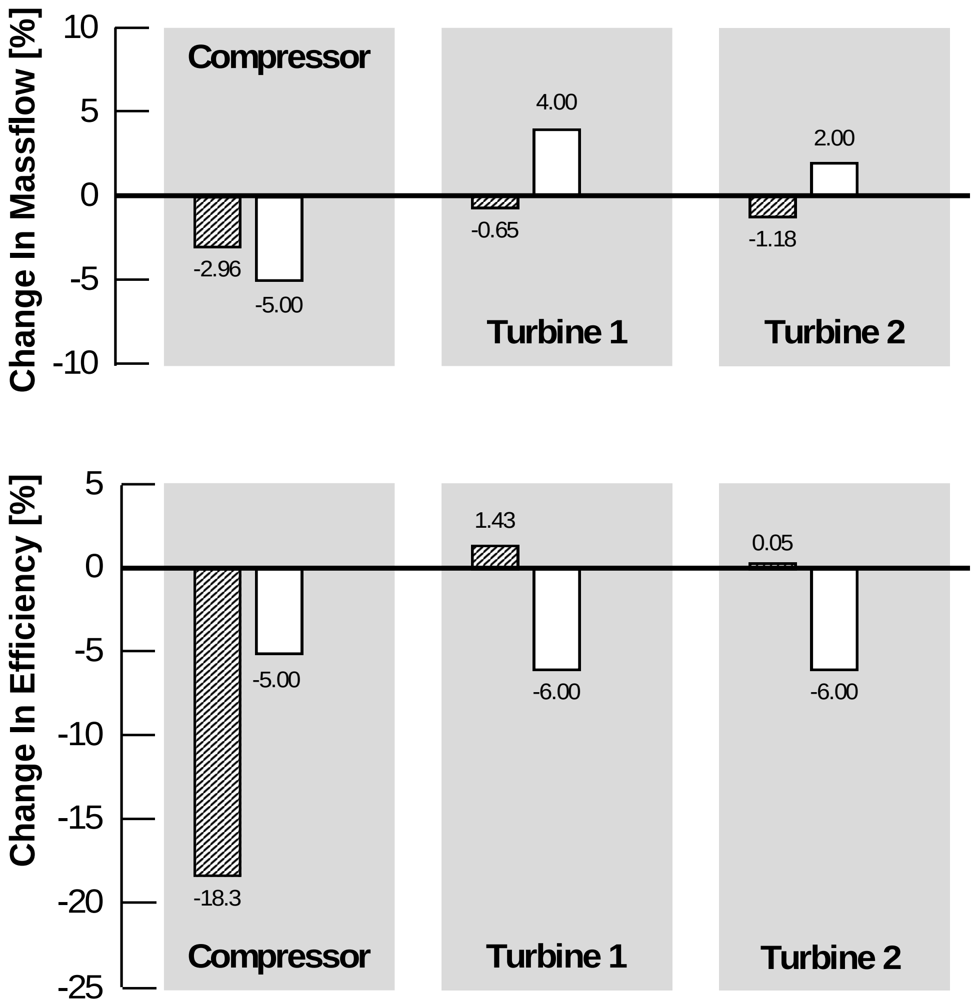
Well defined GPA



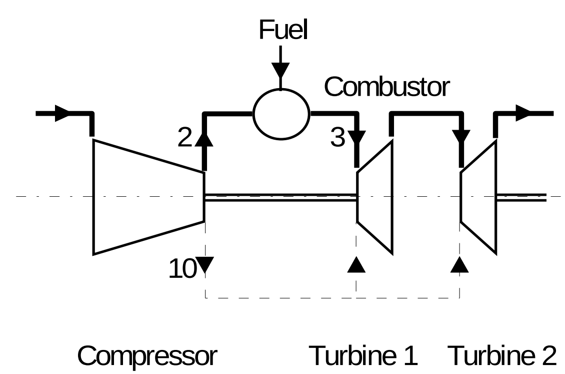
Monitored Parameters (where T is total temperature and P is total pressure):
Rotational Speed of Compressor, P_2, T_2, Fuel Flow, P_4, T_4



Poorly defined GPA



Monitored Parameters (where T is total temperature and P is total pressure):
P_2, T_2, P_{10}, T_{10}, P_3, T_3

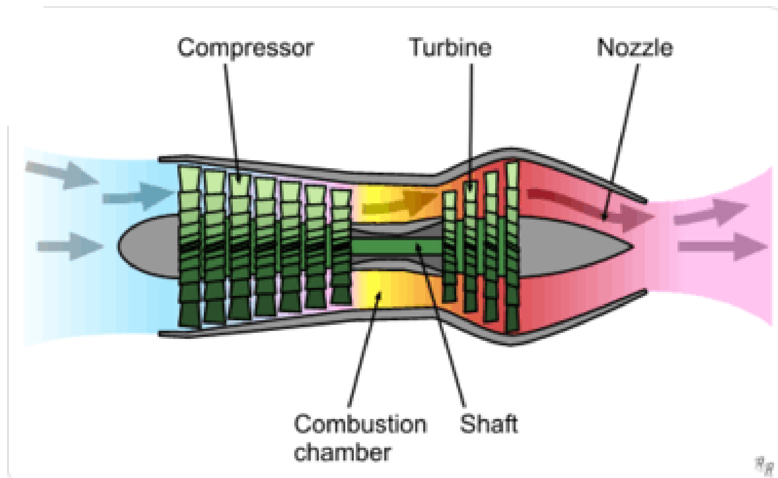


Example of using GPA

Performance analysis of a simple 1-spool Turbojet

```
 ________   ________   ________    ________   ________   ________   ________
|        | |        | |        |  |        | |        | |        | |        |
| INTAKE | | COMPRE | | PREMAS |  | BURNER | | MIXEES | | TURBIN | | NOZCON |
|________| |________| |________|  |________| |________| |________| |________|
1          2          3    |     4          5    |     6          7          8
                           |10                   |10
                           |_____________________|
```

Parameter	DP Value
Altitude	0
Mach number	0
Intake pressure recovery	99%
Relative humidity	60%
Compressor design pressure ratio	8.8
Compressor and Turbine efficiency	89%
Inlet mass flow	77.2 kg/s
TET	1141K
Power setting parameter / handle	PCN



Power setting parameter / Handle: TET

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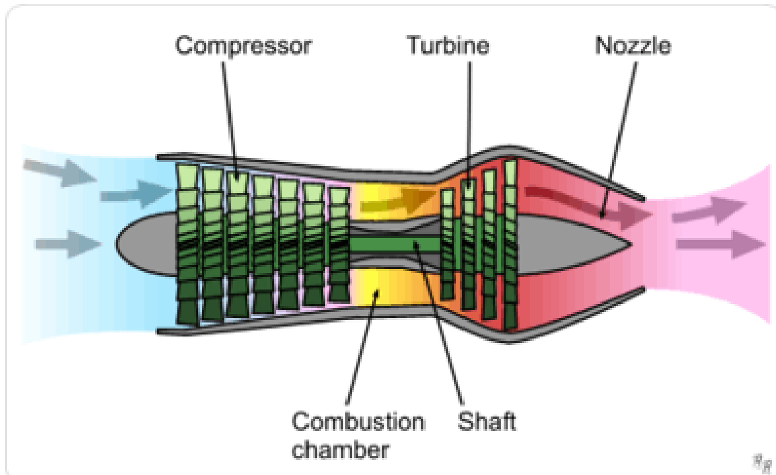
Example of using GPA

Faulty Component: compressor

Health parameters: $\eta_c \quad \Gamma_c$



Measurement parameters: $P_3 \quad T_3$

Example of using GPA

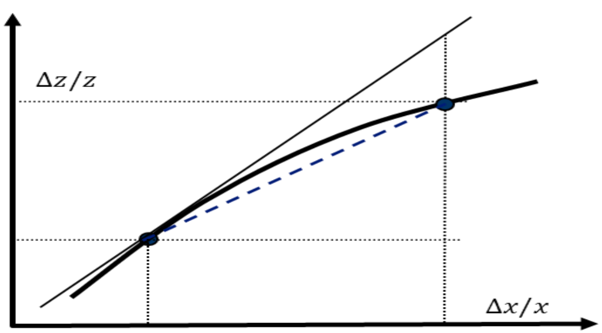
$$\vec{z} = h(\vec{x})$$

$$\Delta\vec{z} = H \cdot \Delta\vec{x} \quad \Rightarrow \quad \Delta\vec{x} = H^{-1} \cdot \Delta\vec{z}$$

- **Step 1: Calculate ICM – *H***
- **Step 2: Calculate FCM – H^{-1}**
- **Step 3: Calculate measurement deviation**
- **Step 4: Calculate health parameter deviation**

Example of using GPA – Step 1

$$H = \left.\frac{\partial \vec{z} / \vec{z}}{\partial \vec{x} / \vec{x}}\right|_0 = \begin{pmatrix} \dfrac{\Delta P_3 / P_3}{\Delta \Gamma_c / \Gamma_c} & \dfrac{\Delta P_3 / P_3}{\Delta \eta_c / \eta_c} \\[2ex] \dfrac{\Delta T_3 / T_3}{\Delta \Gamma_c / \Gamma_c} & \dfrac{\Delta T_3 / T_3}{\Delta \eta_c / \eta_c} \end{pmatrix}_0$$



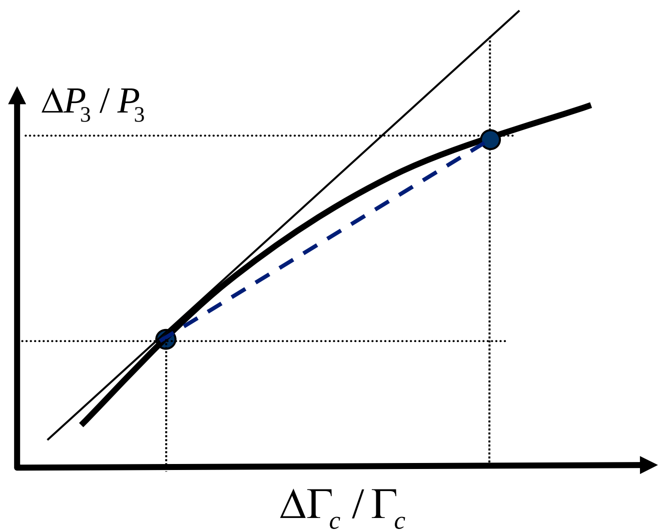
Example of using GPA – Step 1

$$\frac{\Delta P_3 / P_3}{\Delta \Gamma_c / \Gamma_c}$$

-1%

PYTHIA Engine Model

$$\vec{z} = h(\vec{x})$$



Example of using GPA – Step 1

Clean Measurement

Clean engine	Compressor		
	Δηc/ηc	ΔΓc/Γc	ΔPRc/PRc
	0.0	0.0	0.0
Measurement	P3	T3	
	8.69212	561.62	

Degraded Measurement:

(-1% in ΔΓc)

Degraded engine	Compressor		
	Δηc/ηc	ΔΓc/Γc	ΔPRc/PRc
	0.0	-0.01	-0.01
Measurement	P3	T3	
	8.57235	559.32	

$$\frac{\Delta P_3 / P_3}{\Delta \Gamma_c / \Gamma_c} = \frac{(8.57235 - 8.69212)/8.69212}{-0.01} = 1.377915$$

Example of using GPA – Step 1

Clean Measurement:

Clean engine	Compressor		
	$\Delta\eta c/\eta c$	$\Delta\Gamma c/\Gamma c$	$\Delta PRc/PRc$
	0.0	0.0	0.0
Measurement	P3	T3	
	8.69212	561.62	

Degraded Measurement:

(-1% in $\Delta\eta$c)

Degraded engine	Compressor		
	$\Delta\eta c/\eta c$	$\Delta\Gamma c/\Gamma c$	$\Delta PRc/PRc$
	0.0	-0.01	-0.01
Measurement	P3	T3	
	8.57235	559.32	

$$\frac{\Delta T_3/T_3}{\Delta\Gamma_c/\Gamma_c} = \frac{(559.32 - 561.62)/561.14}{-0.01} = 0.40953$$

Example of using GPA – Step 1

$$\boldsymbol{H} = \begin{pmatrix} 1.377915 & -0.7401 \\ 0.40953 & -0.70332 \end{pmatrix}$$

Gas Path Analysis (GPA) – Step 2

$$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$H^{-1} = \frac{1}{|H|} \cdot \boldsymbol{adj\ H}$$

Determinant of H: $|H| = a \cdot d - c \cdot b$

Adjoint of H: $adj\ H = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Example of using GPA – Step 2

$$
\begin{array}{ccc}
 & a & b \\
\boldsymbol{H} = & \left(\begin{array}{cc} 1.377915 & -0.7401 \\ 0.40953 & -0.70332 \end{array}\right) & \\
 & c & d
\end{array}
$$

Determinant of H: $|H| = a \cdot d - c \cdot b$

$|H| = 1.377915*(-0.70332) – 0.40953*(-0.7401)$
$= -0.666027$

Example of using GPA – Step 2

$$H = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1.377915 & -0.7401 \\ 0.40953 & -0.70332 \end{pmatrix}$$

Adjoint of H:

$$adj\ H = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\boldsymbol{adj\ H} = \begin{pmatrix} -0.70332 & 0.7401 \\ -0.40953 & 1.377915 \end{pmatrix}$$

(replacing each in ***H*** with its cofactor and transposing the result)

Example of using GPA – Step 2

Inverse of H:

$$H^{-1} = \frac{1}{|H|} \cdot adjH$$

$$H^{-1} = \frac{1}{-0.666027} * \begin{pmatrix} -0.70332 & 0.7401 \\ -0.40953 & 1.377915 \end{pmatrix}$$

$$H^{-1} = \begin{pmatrix} 1.055996 & -1.11121 \\ 0.614884 & -2.06886 \end{pmatrix}$$

Example of using GPA – Step 3

Linear engine GPA model:

$$\begin{pmatrix} \Delta\Gamma_c/\Gamma_c \\ \Delta\eta_c/\eta_c \end{pmatrix} = \begin{pmatrix} 1.055996 & -1.11121 \\ 0.614884 & -2.06886 \end{pmatrix} * \begin{pmatrix} \Delta P_3/P_3 \\ \Delta T_3/T_3 \end{pmatrix}$$

$$\Delta\vec{x} = H^{-1} \cdot \Delta\vec{z}$$

Example of using GPA – Step 3

Clean Measurement:

Clean engine	Compressor		
	$\Delta\eta c/\eta c$	$\Delta\Gamma c/\Gamma c$	$\Delta PRc/PRc$
	0.0	0.0	0.0
Measurement	P3	T3	
	8.69212	561.62	

Degraded Measurement:

Degraded engine	Compressor		
	$\Delta\eta c/\eta c$	$\Delta\Gamma c/\Gamma c$	$\Delta PRc/PRc$
	?	?	?
Measurement	P3	T3	
	8.386	558.36	

$$\begin{bmatrix} \dfrac{\Delta P_3}{P_3} \\ \dfrac{\Delta T_3}{T_3} \end{bmatrix} \times 100 \quad ?$$

$$\begin{pmatrix} \Delta P_3/P_3 \\ \Delta T_3/T_3 \end{pmatrix} *100 = \begin{pmatrix} \dfrac{8.386-8.69212}{8.69212} \\ \dfrac{558.36-561.62}{561.62} \end{pmatrix} *100 = \begin{pmatrix} -3.52181 \\ -0.58046 \end{pmatrix}$$

Example of using GPA – Step 4

$$\begin{bmatrix} \dfrac{\Delta\Gamma_c}{\Gamma_c} \\ \dfrac{\Delta\eta_c}{\eta_c} \end{bmatrix} \times 100 = H^{-1} \cdot \begin{bmatrix} \dfrac{\Delta P_3}{P_3} \\ \dfrac{\Delta T_3}{T_3} \end{bmatrix} \times 100$$

Example of using GPA – Step 4

$$\Delta\vec{x} = H^{-1} \cdot \Delta\vec{z}$$

$$\begin{pmatrix} \Delta\Gamma_c/\Gamma_c \\ \Delta\eta_c/\eta_c \end{pmatrix} = \begin{pmatrix} 1.055996 & -1.11121 \\ 0.614884 & -2.06886 \end{pmatrix} * \begin{pmatrix} -3.52181 \\ -0.58046 \end{pmatrix} = \begin{pmatrix} -3.074 \\ -0.96461 \end{pmatrix} \%$$

$$RMS = \{[(-3+3.074)^2 + (-1+0.96461)^2]/2\}^{0.5} = 0.0580039$$